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# FERTILITY PROJECTIONS BY THE COHORT METHOD FOR CANADA – 1969-84

by

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# FOR CANADA -- 1969-84

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TABLE OF CONTENTS



LIST OF TABLES

## LIST OF CHARTS

# PROJECTIONS DÉMOGRAPHIQUES PAR LA MÉTHODE D'ANALYSE PAR COHORTE*

## CANADA — 1969-84


## Résumé

Les projections démographiques du B.F.S. en 1969 reposent sur la méthode des composantes. Les séries de naissances tirées des projections des taux de fécondité par la méthode de la fécondité de la cohorte forment une des composantes. Le présent ouvrage traite des aspects techniques de ces projections de la fécondité.

Les principales opérations permettant d'établir les projections des naissances par la méthode d'analyse par cohorte sont les projections de la descendance finale et de la distribution de la fécondité par âge, ainsi que la combinaison de ces deux composantes pour établir les taux spécifiques de fécondité par âge d'une cohorte. D'autres méthodes, comme celles des taux en chaîne, de l'ajustement de la courbe de Gompertz et de l'extrapolation graphique des taux spécifiques de fécondité d'une cohorte déjà établis, ont été utilisées pour les projections des taux finals de natalité des cohortes avec descendance inachevée. La projection de taux finals de natalité des cohortes qui doivent entrer dans leur période de reproduction au cours de la période de projection se fonde sur des séries chronologiques associées à une distribution paritaire. Les séries de distribution paritaire offrent quelques avantages analytiques et offrent une perspective temporelle plus claire que les séries chronologiques sur les taux de fécondité générale.

L'analyse des séries chronologiques sur la nuptialité, la distribution paritaire et l'espacement des naissances donne d'intéressants résultats sur lesquels on peut établir la projection des courbes par âge de la période de reproduction. L'ouvrage traite de la manière dont les hypothèses relatives à la descendance finale ont été combinées avec les hypothèses relatives à la distribution de la fécondité par âge pour produire les taux spécifiques de fécondité par âge d'une cohorte. Enfin, il soulève le problème de la mise en concordance entre les séries du passé et les séries projetées.



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# FERTILITY PROJECTIONS BY THE COHORT METHOD*

# FOR CANADA — 1969-84


## Abstract

The 1969 DBS population projections are based on the component method. One of the components are the births series derived from fertility rates projected by the cohort fertility approach. This paper deals with the technical aspects of these fertility projections.

The main operations involved in projecting births by the cohort approach are the projections of completed fertility and that of age pattern of fertility as well as the combination of these two components in order to generate the cohort-age-specific fertility rates. Alternative methods such as the chain ratio, fitting the Gompertz curve, and graphical extrapolation of past cohort-parity-age-specific fertility rates were applied to project the final birth rates for the cohorts with incomplete fertility. The projection of final birth rates for the cohorts due to enter childbearing later in the projection period is based on time series related to parity distribution. The parity distribution series offer some analytical advantages and have a clearer time perspective than the time series on general fertility rates.

Analysis of time series concerning nuptiality, parity distribution and childspacing revealed interesting features on which to base the projection of age patterns of childbearing. The paper discusses the ways of associating the assumed levels of completed fertility rates with assumed age patterns of fertility to generate cohort-age-specific fertility rates. Finally some problems of reconciliation of past and projected series are raised.



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## 1. Introduction

The 1969 DBS population projections constitute the first attempt to use a cohort instead of a period approach for projecting fertility rates for Canada. A short description of the cohort method was included in the summary report of these projections (Canada, 1970). The object of the present report is to give a detailed account of the procedures in the application of the cohort method to the Canadian data. It is hoped that this report will be of help to users who are interested in understanding the procedures used to arrive at the final figures, and to those who wish to carry out further research in the field of population projections.

Despite notable progress in the techniques of population projections in recent years, demographic forecasting still relies largely on the projector's own intuition regarding future trends and on the straightforward extrapolation of past trends (see Akers, 1965; Beshers, 1965; Goodman, 1968; Keyfitz, 1964; Bumpass and Westoff, 1969; Siegel and Akers, 1969). Demographers are still a long way from achieving the highly formalized types of models and the statistical sophistication of present-day econometrics. Yet, it is toward the development of such formal models that professionals engaged in population forecasting should direct their efforts if demographic projections are to acquire full recognition as a scientific exercise. Some of the substantive and methodological results obtained in the course of preparatory work might ultimately serve as ingredients for the construction of a formal model, and this paper is a documentation of these results.

The main steps in projecting births by the cohort method can be summarized as follows:

1. Projection of the completed fertility rate, expressed as the number of children per woman upon completion of the childbearing span, i.e. 49 years of age, for the cohorts of women of childbearing age during the projection period (1969-1984).

2. Projection of the age pattern of fertility, expressed as the percentage distribution of fertility by age for each of the above cohorts.

3. Multiplication of the assumed completed fertility rates by the assumed percentage distribution of fertility by age, for generating cohort age-specific fertility rates.

4. Translation of the cohort into period age-specific fertility rates.

5. Multiplication of the period age-specific fertility rates by the mid-year female survivors for corresponding ages and addition of the products thus obtained for deriving the annual number of births required for the population projection.

The most complex problems arise in the first three steps. Therefore, the main effort of this paper will be toward a discussion of these problems. The procedures involved in the two remaining steps are conventional and will be given only cursory attention. Before discussing each of the steps, a brief discussion of why the cohort method was preferred to the period method will be presented. This entails a consideration of the general conceptual differences between the two approaches as well as the specific problems arising from peculiarities in the Canadian data. The achievement

of internal consistency within the series projected by the cohort method poses difficulties and will be given due consideration.

## 2. Cohort versus Period Method

The cohort method and the period method are two approaches that can be adopted in analysing changes in fertility rates. The first consists of following the changes longitudinally in terms of successive birth generations (cohorts). The second consists of following these changes cross-sectionally in terms of successive periods of time. The Lexis diagram in Chart 1 illustrates the two approaches. If $m_x$ is the notation for the age-specific fertility rate by cohort, then the sum of the $m_x$ values gives the total number of births per woman of the cohort over the entire childbearing span. If $f_x$ is used to denote the age-specific fertility rate observed at a point in time, the sum of $f_x$ gives the total number of births per woman which occur to the cohorts in the childbearing age at this point in time. The latter may also be defined as the number of children that would be born to a hypothetical cohort of women if they experienced no mortality and were subjected to the same schedule of age-specific fertility rates, $f_x$, as observed during a given calendar year.(1) To avoid confusion, the term "Completed Fertility Rate" (CFR) will be used throughout this text to refer to the total childbearing performance of a real cohort and the term "Total Fertility Rate" (TFR) to refer to the same performance of a hypothetical cohort.

Thus, for the real cohort born y-x years ago:

$$CFR_{(y-x)} = \sum_{x=15}^{49} m_x$$

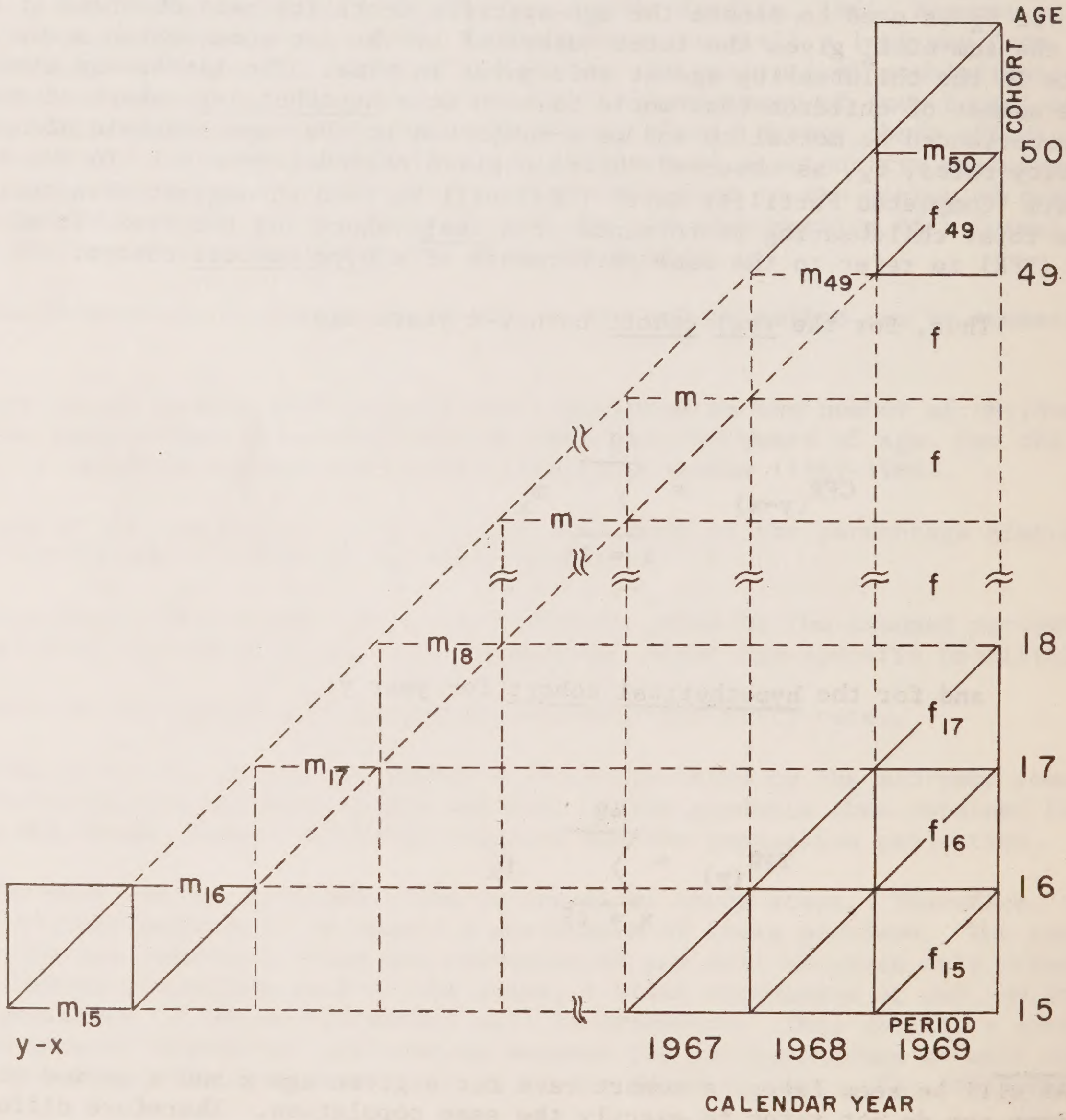
and for the hypothetical cohort for year y:

$$TFR_{(y)} = \sum_{x=15}^{49} f_x$$

(1) As will be seen later, a cohort rate for a given age x and a period rate for the same age do not refer to exactly the same population. Therefore different notations are used for the two rates.

CHART I

## LEXIS DIAGRAM SHOWING THE AGE-SPECIFIC FERTILITY RATES IN PERIOD (VERTICAL) AND COHORT (DIAGONAL) SET-UP



From the point of view of projectional efficiency, the cohort approach can be credited with two particular advantages. One such advantage stems from the possibility of capitalizing on the previous experience of real cohorts to predict the likelihood of subsequent events. The extent to which this can be achieved depends, of course, upon the stage of family formation that a cohort has reached at a given moment and upon the adequacy of the information on related reproductive measurements. One can indeed see the life axis of a generation as the foci of a succession of events — marriage, first birth, second birth, divorce, remarriage, and so on until completion of the reproductive span. The availability of information with regard to reproductive events, such as parity and childspacing, would greatly increase the ability to predict remaining fertility. Unfortunately, specifications in such detail are either lacking or are not available in the proper format for Canada. The data on age-specific and cumulative fertility rates by cohort are available, and a reliance on these minimum statistics is the cornerstone of our approach in deriving remaining fertility for cohorts with incomplete childbearing experience.

The second advantage is of a more general analytical character. Fertility may be characterized by its quantity and by its timing or, in other words, by its level as measured by the total or completed fertility rate, and by its age pattern (distribution of births by age of mother). When the period framework of observation is employed, neither the latter nor the former can be directly identified with respect to relevant cohorts, their effects become intermingled (Ryder, 1969). Temporal variation in the total fertility rate is the result of the joint effect of changes in both the family size and the age pattern of fertility of the cohorts in a particular period. Recent demographic history has been dominated by marked shifts in these variables. The post-war baby boom is a case in point. The dramatic upsurge of the total fertility rate during that period is due to the combined effect of making up for earlier postponements together with earlier childbearing, and the achieving of larger families than during the pre-war years. In order to trace the origin of these fluctuations in the period total fertility rate, and by the same token, in its age variation, one must resort to cohort analysis. In other words, the latter is required for an adequate understanding of the immediate factors of temporal variations in period fertility rates. In using the cohort approach, projections can be made for real cohorts separately in terms of family size (completed fertility), and in terms of the age pattern of fertility.

To conclude this section, a word regarding the data problems arising from the adoption of the cohort approach is in order. Births for Canada are recorded and published by age of mother at delivery, and not by date of birth of the mother. The latter information is required for calculating the cohort age-specific fertility rates. These were obtained by reading diagonally from a table in which rates are cross-classified both by age of mother and by calendar year of observation. This procedure is tantamount to equating the birth rate in the age interval x to x+1, observed in a calendar year y, with the birth rate in the same age interval of the cohort y-x. However, the period birth rate in the x+1 age interval reflects the childbearing performance not of one but of three adjacent cohorts, y-x-1, y-x and y-x+1. Assuming a uniform distribution of births through time, the middle cohort contributes 74.3 per cent, the adjacent younger one 17.0 per cent, and the adjacent older one 8.7 per cent to the calendar-year births of a given age interval.(2) These cohorts may differ among

(2) This percentage breakdown is due to the fact that in Canada the annual birth rate for a given age is calculated by dividing the number of births registered during the calendar year occurring to the women x years old at delivery by the number of women in that age as of June 1 instrad of midyear (July 1).

themselves either in terms of their respective number, their respective reproductive performance or both. No adjustment for these differentials has been attempted in converting period rates into cohort rates, nor was it possible to take into account the cohort differentials with respect to their origin (immigrants and natives), or marital status. Proper adjustments for such differentials would have greatly enhanced the analytical rigour of the cohort method.

## 3. Projecting Completed Cohort Fertility

Women who will bear children at some time during the projection period, 1969-1984, belong to the cohorts born between 1920 and 1969. In 1969, at the beginning of the projection period, these women range in age from zero to 49 years. The youngest ones having still a long way to go before entering childbearing while the older ones are at different stages of the family building process and, hence, will require different treatment in arriving at the final fertility rate. For the convenience of analysis four groups of cohorts were formed as follows:

I. Women born between 1920 and 1939 who were at least 30 years old in 1969 and who, as such, have passed the peak of reproduction.

II. Women born between 1939 and 1949 who are 20 to 30 years old.

III. Women born between 1949 and 1961 who have just entered or will enter childbearing only during the projection period.

IV. Women born between 1961 and 1969 who will begin reproduction only in the later part or projection period.

Three alternative methods, namely the chain ratio, the Gompertz function, and the graphical extrapolation of cohort-age-specific fertility rates by birth order, are used for projecting fertility rates for the first group of cohorts. For the second group, rates are obtained by combining the actual fertility to date with the assumptions concerning the over-all age distribution of fertility. For the third group, use is made of parity distribution as a framework for projecting the completed fertility rate. As for the last group, fertility is assumed to remain constant at the levels of the 1960-61 cohort as projected by the parity distribution method.

### 3.1 Chain Ratio Method

As a straightforward arithmetical extrapolation, the chain ratio procedure is flexible enough to apply to a number of situations in which it can be assumed that the ratios of fertility rates will remain relatively stable over the projected period. Let $f_x^y$ and $f_{(x+1)}^y$ be the cumulative fertility rate up to ages x and x+1 for the cohort y, and let $f_x^{(y+1)}$ and $f_{(x+1)}^{(y+1)}$ be the same measurements for the next youngest

cohort, y+1, then, in order to estimate $f_{(x+1)}^{(y+1)}$, given $f_x^y$, $f_{(x+1)}^y$, and $f_x^{(y+1)}$,

$$f_{(x+1)}^{(y+1)} = \frac{f_{(x+1)}^{y}}{f_x^y} \cdot f_x^{(y+1)} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

The procedure, when repeated for each cohort and successive ages up to 50, takes the form of a chain of ratios. Its weakness is that, for any given age, a constant ratio is assumed to apply to the whole projection period. In the event of a relative decline in fertility from one cohort to the next, the procedure tends to overstate the projected values, and to understate them when fertility is rising.

One way to overcome this difficulty is to build a correction factor into the model to compensate for bias due to fertility trends. This may be done by taking into account the changes between the corresponding ratios from two previous cohorts for which the values are available, and applying them to the ratio that is involved in estimating $f_{(x+1)}^{(y+1)}$, assuming linear change from one cohort to the next (Nagnur, 1970). Thus:

Let $r_{(y-1)} = \dfrac{f_{(x+1)}^{(y-1)}}{f_x^{(y-1)}}$ and $r_y = \dfrac{f_{(x+1)}^{y}}{f_x^y}$

Then the change equals $r_y - r_{(y-1)}$

and taking the change into consideration, $f_{(x+1)}^{(y+1)}$ may be estimated as

$$f_{(x+1)}^{(y+1)} = \left[ \frac{f_{(x+1)}^{y}}{f_x^y} + r_y - r_{(y-1)} \right] f_x^{(y+1)} \quad \ldots\ldots\ldots\ldots\ldots (1a)$$

i.e.

$$f_{(x+1)}^{(y+1)} = \left[ 2r_y - r_{(y-1)} \right] f_x^{(y+1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Unfortunately, a correction factor assuming a linear change implicit in the above formula can, as the author admits, lead to absurd results when applied over a longer period of time. Under the circumstances, a graphical adjustment of values derived by the chain ratio formula without a correction factor was deemed preferable. The operation was performed in the following way: the cumulative rates were derived by formula 1 then, by simple differentiation, the age-specific rates were obtained and plotted on the graph. A free-hand correction was made to ensure a smooth transition from actual to projected values. The resulting corrected values are slightly lower than the original ones.

### 3.2 Projection by the Gompertz Function

In order to project the remaining and final fertility rates for a given cohort by the chain ratio method, the appropriate estimator had to be derived from the series for previous cohorts. A method that does not require reference to other cohorts and that relies solely on evidence embedded in the cohort itself, is the Gompertz function which will be presented briefly here.

A number of studies have revealed that the actual cumulative fertility rate can be approximated by a curve of a logistic, asymmetrical form, best expressed by the Gompertz formula (Romaniuk and Tanny, 1969).

$$Y = KG^{B^{X}}$$

The cumulative fertility rate, Y, at each successive age, X, can be estimated by solving the above formula for the constants K, G, and B.

The following equations permit one to solve for the constants K, G, and B.

$$B^{r} = \frac{\log Y_2 - \log Y_1}{\log Y_1 - \log Y_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\log G = \frac{\log Y_1 - \log Y_0}{B^{r} - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

$$\log K = \log Y_0 - \log G \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

If one is primarily interested, as is the case here, in the final cumulative fertility rate, K, the following formula is appropriate:

$$\log K = \frac{(\log Y_0)\ (\log Y_2) - (\log Y_1)^2}{(\log Y_0 + \log Y_2 - 2 \log Y_2} \quad \ldots\ldots\ldots\ldots (6)$$

$Y_0$, $Y_1$ and $Y_2$ are cumulative fertility rates for the lower, middle and upper selected ages, respectively; K is the asymptotic value of the function corresponding here to the total fertility rate. Selection of the three age-points is such as to produce an equidistant r difference:

$$X_2 - X_1 = X_1 - X_0 = r$$

This selection is crucial in achieving the best agreement between actual and derived values. Tests that have been performed on actual cohorts with completed fertility and hypothetical cohorts with a number of age combinations have revealed that the best agreement is obtained by the set of ages which include 18, 25 and 32. Therefore, this set is used to derive the final fertility by formula (6).

The actual values and those derived by formula (6) are shown in Table 1.

3.3 <u>Graphical Extrapolation of the Age-Specific Fertility Rates by Birth Order</u>

For the cohorts of women who have passed the peak of procreation, the age-specific fertility rates completed to date for each individual <u>birth order</u> have been plotted on the graph, and the remaining fertility has been graphically extrapolated. There is an analytical advantage in drawing on birth order-age-specific rates instead of general rates. Analysis of inter-cohort combined with that of inter-order variations may indeed reveal certain interesting features upon which to base the projection of the remaining fertility. In addition, because of further developments which will be discussed later, analysis by birth order or parity was necessary for estimating of the age pattern of childbearing and childspacing.

Table 2 presents the completed fertility rates obtained by the three methods described above. The highest values are obtained by the Gompertz formula, the lowest by graphical extrapolation of the order age-specific fertility rates, while the chain ratio yields an intermediate series. Except for the last few cohorts, deviations are small and for practical purpose can be disregarded.(3)

On theoretical grounds, the Gompertz function seems to be the method best suited to handle the kind of projections we are faced with here. The results are predetermined by the logic of the relationship implied in the mathematical model. The method is self-sufficient, in the sense that the required data are internal to the cohort itself. However, it was felt that further experimentation was necessary to bring the method to a degree of perfection that could ensure its full success. In particular, the method is weak in predicting rates at the upper ages of the reproductive span, although it does eventually yield acceptable final fertility rates (asymptotic values) (Romaniuk and Tanny, 1969).

In any event, since there is no way of knowing which of the three procedures is the best in projecting the final fertility rate, and since the obtained series are

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(3) It should be mentioned that even for cohorts with completed fertility, the total fertility rate obtained by the summation of the total fertility rates by birth order, are for some obscure reasons, lower by almost 2 per cent than those calculated directly from the birth data furnished by Vital Statistics. If this underestimation (or over-estimation) is accounted for, then the difference between the derived series would actually be smaller.

TABLE 1. Completed Fertility Rates, Actual and as Derived by Means of the Gompertz Formula

| Year | Derived value (a) | Actual value (b) | (a)/(b) | Year | Derived value (a) | Actual value (b) | (a)/(b) |
|---|---|---|---|---|---|---|---|
| | Cohort rates | | | | | | |
| 1905 ............ | 2,656 | 2,879 | 0.92 | 1915 ........... | 3,002 | 2,885 | 1.04 |
| 1906 ............ | 2,568 | 2,840 | 0.90 | 1916 ........... | 2,957 | 2,879 | 1.03 |
| 1907 ............ | 2,551 | 2,832 | 0.90 | 1917 ........... | 2,948 | 2,928 | 1.01 |
| 1908 ............ | 2,486 | 2,736 | 0.91 | 1918 ........... | 2,925 | 2,893 | 1.01 |
| 1909 ............ | 2,554 | 2,762 | 0.92 | 1919 ........... | 3,323 | 3,231 | 1.03 |
| 1910 ............ | 2,585 | 2,711 | 0.95 | 1920 ........... | 3,321 | 3,267 | 1.02 |
| 1911 ............ | 2,696 | 2,720 | 0.99 | 1921 ........... | 3,282 | 3,286 | 1.00 |
| 1912 ............ | 2,819 | 2,767 | 1.02 | 1922 ........... | 3,213 | 3,221 | 1.00 |
| 1913 ............ | 2,996 | 2,873 | 1.04 | 1923 ........... | 3,244 | 3,258 | 1.00 |
| 1914 ............ | 3,111 | 2,913 | 1.07 | 1924 ........... | 3,286 | 3,291 | 1.00 |
| | Period rates | | | | | | |
| 1926 ............ | 3,099 | 3,356 | 0.92 | 1948 ........... | 3,224 | 3,423 | 0.94 |
| 1927 ............ | 3,041 | 3,319 | 0.92 | 1949 ........... | 3,275 | 3,438 | 0.95 |
| 1928 ............ | 3,011 | 3,296 | 0.91 | 1950 ........... | 3,308 | 3,433 | 0.96 |
| 1929 ............ | 2,956 | 3,218 | 0.92 | 1951 ........... | 3,358 | 3,480 | 9.96 |
| 1930 ............ | 3,012 | 3,284 | 0.92 | 1952 ........... | 3,495 | 3,621 | 0.97 |
| 1931 ............ | 3,020 | 3,201 | 0.94 | 1953 ........... | 3,549 | 3,702 | 0.96 |
| 1932 ............ | 2,943 | 3,086 | 0.95 | 1954 ........... | 3,610 | 3,812 | 0.95 |
| 1933 ............ | 2,729 | 2,865 | 0.95 | 1955 ........... | 3,589 | 3,817 | 0.94 |
| 1934 ............ | 2,690 | 2,804 | 0.96 | 1956 ........... | 3,659 | 3,849 | 0.95 |
| 1935 ............ | 2,658 | 2,754 | 0.97 | 1957 ........... | 3,732 | 3,929 | 0.95 |
| 1936 ............ | 2,586 | 2,695 | 0.96 | 1958 ........... | 3,707 | 3,884 | 0.95 |
| 1937 ............ | 2,516 | 2,645 | 0.95 | 1959 ........... | 3,777 | 3,947 | 0.96 |
| 1938 ............ | 2,531 | 2,701 | 0.94 | 1960 ........... | 3,742 | 3,910 | 0.96 |
| 1939 ............ | 2,457 | 2,653 | 0.93 | 1961 ........... | 3,687 | 3,857 | 0.96 |
| 1940 ............ | 2,570 | 2,760 | 0.93 | 1962 ........... | 3,606 | 3,773 | 0.96 |
| 1941 ............ | 2,640 | 2,824 | 0.93 | 1963 ........... | 3,515 | 3,690 | 0.95 |
| 1942 ............ | 2,804 | 2,954 | 0.95 | 1964 ........... | 3,365 | 3,521 | 0.96 |
| 1943 ............ | 2,906 | 3,030 | 0.96 | 1965 ........... | 3,039 | 3,163 | 0.96 |
| 1944 ............ | 2,855 | 3,000 | 0.95 | 1966 ........... | 2,705 | 2,826 | 0.96 |
| 1945 ............ | 2,807 | 3,005 | 0.93 | 1967 ........... | 2,496 | 2,593 | 0.96 |
| 1946 ............ | 3,142 | 3,356 | 0.94 | 1968 ........... | 2,404 | 2,445 | 0.98 |
| 1947 ............ | 3,341 | 3,575 | 0.93 | 1969 ........... | 2,369 | 2,410 | 0.98 |

TABLE 2. Projected Completed Fertility Rates for Cohorts with Childbearing Experience

(per 1,000 women 49 years of age)

| Cohort | Age in 1969 | Gompertz formula | Chain ratio | Graphical extrapolation by birth order | (3)/(4) | (5)/(4) |
|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 1920-21 | 48 | 3,321 | 3,267 | 3,151 | 1.017 | .964 |
| 1921-22 | 47 | 3,282 | 3,286 | 3,202 | .999 | .974 |
| 1922-23 | 46 | 3,213 | 3,220 | 3,167 | .998 | .983 |
| 1923-24 | 45 | 3,244 | 3,258 | 3,194 | .996 | .980 |
| 1924-25 | 44 | 3,286 | 3,289 | 3,204 | .999 | .974 |
| 1925-26 | 43 | 3,280 | 3,272 | 3,201 | 1.008 | .978 |
| 1926-27 | 42 | 3,262 | 3,254 | 3,181 | 1.002 | .978 |
| 1927-28 | 41 | 3,348 | 3,303 | 3,217 | 1.014 | .974 |
| 1928-29 | 40 | 3,388 | 3,287 | 3,202 | 1.031 | .974 |
| 1929-30 | 39 | 3,559 | 3,423 | 3,319 | 1.040 | .970 |
| 1930-31 | 38 | 3,558 | 3,417 | 3,312 | 1.041 | .969 |
| 1931-32 | 37 | 3,549 | 3,414 | 3,288 | 1.040 | .963 |
| 1932-33 | 36 | 3,446 | 3,327 | 3,208 | 1.036 | .964 |
| 1933-34 | 35 | 3,314 | 3,232 | 3,052 | 1.025 | .944 |
| 1934-35 | 34 | 3,241 | 3,213 | 2,982 | 1.009 | .928 |
| 1935-36 | 33 | 3,165 | 3,185 | 2,918 | .994 | .916 |
| 1936-37 | 32 | 2,979 | 3,067 | 2,862 | .971 | .933 |
| 1937-38 | 31 | 2,921 | 3,058 | 2,823 | .955 | .923 |
| 1938-39 | 30 | | 2,995 | | | |

of a very narrow range, the intermediate series derived by the chain ratio method has been selected to represent completed fertility of cohorts 1920-21 to 1938-39.

### 3.4 Combining Actual Fertility to Date with Assumptions about the Over-all Age Distribution of Fertility

Although the cohorts to which this method applies are only 20 to 30 years old (1939-1949), they have accumulated a reproductive experience that is worthy of consideration as basis for projecting completed fertility. Crucial in this approach is the use of a set of assumptions regarding the age pattern of fertility expressed as the percentage distribution of the birth rates which these cohorts are expected to approximate upon completion of their childbearing. The completed fertility rate is obtained by applying the assumed percentage distribution of fertility to the actual fertility rates that have been completed up to the projection date.

Cohorts examined here exhibit an unprecedented drop in fertility. For example, the cumulative fertility rate up to age 25 was 20 per cent less for the 1942-43 cohort than that for the 1939-40 cohort. The 1948-49 cohort has achieved a cumulative fertility up to age 25 of 30 per cent less than the 1939-40 cohort up to the same age. In making an assumption as to the completed fertility rate for these cohorts, the fundamental question is to know whether the observed decline in fertility is indicative of a shift in family size norms or in the timing of childbearing. Opinions on this question among demographers are divergent.

According to one set of opinions, the phenomenon is essentially that of a shift in the timing of births from younger to older ages, a shift which will have no marked effect on the completed fertility rate. The same family size as before will be achieved, although through a different calendar schedule. One can think of a number of factors that might have caused such a shift in timing. First, as far as can be inferred from the available data, the nuptiality rate seems to have undergone a decline. For example, the cumulative nuptiality rate up to age 20 has fallen from 45 per cent for the 1939-40 cohort, to 39 per cent for the 1944-45 cohort (see Chart 4). One is tempted to interpret this as being indicative of the emergence of later marriage, resulting in later maternity. Second, it has been argued that people have a less optimistic perception of economic prospects now than in previous years. This may have induced young couples to plan more carefully their family formation by delaying pregnancies. Third, due to the availability of more efficient contraceptives some prenuptial conceptions which were prevalent in the past may have been avoided.

Views consistent with this thesis have been expressed by a number of demographers on one occasion or another (Biraben and Légaré, 1967; Pressat, 1969; Ryder and Westoff, 1969). Surveys conducted among American women on their fertility expectations seem to support the view that changes are primarily structural and not quantitative. No significant variations in desired family size has been revealed at an aggregate level by the surveys carried out in 1955, 1960 and 1965.

Other demographers (Calot, Hémery and Piro, 1969) hold a different view of what is the real meaning of the recent decline of fertility. For them, this decline is a reflection of either a fundamental change in the attitude toward family size, or the emergence of a reproductive pattern that will ultimately result in a lower completed fertility rate. Couples will either desire fewer children or be forced to limit pregnancies as a result of a more pessimistic appraisal of the economic climate. A conspicuous feature of the recent decline in fertility is that it affects practically all industrialized countries, all generations and all parities. However, the decline was

sharper in Canada and the U.S.A., where the initial fertility level was higher than in other industrialized countries. As for the parities, the decline is proportionately greater among couples with two and more children than among those with one child. In the light of these observations one is inclined to see the process as being a further accentuation of the tendency toward a homogeneous reproductive pattern through a downward readjustment of family size among the populations of industrialized countries.

The present writer is more inclined to interpret the recent decline in fertility as being indicative of a process that will end in smaller family size (either because fewer offspring are desired or imposed by circumstances) rather than as a shift toward an older childbearing pattern. This conclusion was reached after having examined the past trends and projected future trends in the age patterns of childbearing. The reader will find this topic discussed more extensively in the next section. It suffices here to state the two basic assumptions regarding the expected future trends in the fertility age structure: (i) that the prevailing downward trend of childbearing will be further accentuated; and (ii) that a progressive shift toward more central reproductive ages from both ends of the childbearing age spectrum will occur. The first can be considered as a younger and the second as an older age pattern assumption. In no case do we foresee an upward shift in the age structure of childbearing that would be consistent with a total fertility of, say, 3.4 children per woman as achieved by the 1929-32 cohort, or even the rate of 3.0 as achieved by the 1938-39 cohort. In other words, we do not anticipate such an increase in fertility at the older ages as to make up for the losses at younger ages experienced by the 1939-49 cohorts.

The remaining operation consists of dividing, for each cohort, the actual fertility rate completed to a given age by the projected percentage distribution of fertility up to the corresponding age. The resulting values provide a zone within which the true values of completed fertility for the cohorts born between 1939 and 1949 are likely to fall. They are shown in Table 3.

### 3.5 Parity Distribution as a Framework for Projecting Completed Fertility for Cohorts Entering Childbearing During the Projection Period

In the case of cohorts which at the beginning of the projection had reached a more or less advanced stage of family formation, it was possible to capitalize on their childbearing experience to date for making assumptions about their remaining and final fertility rates. Dependence upon incomplete cohort fertility data is the saliant feature of the procedures discussed so far. A different situation arises when dealing with cohorts which are due to enter childbearing later on during the projection period. Here, instead of making use of evidence "internal" to the cohorts for which the completed fertility rate is being sought, one has to resort to "external" evidence which is borrowed from the experience of older cohorts. Two avenues are open for accomplishing this with the available data. One consists of making use of time series of completed fertility rates (4) to extrapolate the future movement. The other one is to draw upon parity distribution data, i.e. completed fertility rates for each <u>individual birth order</u> (parity). The latter approach has the advantage of being an analytical one, since each parity can be assessed as a function of certain variables. Fertility for the first order, for example, may be related to childlessness and its determinants, nuptiality and sterility. Moreover, a look at historical series will reveal some definite trends for certain parities that suggest the likely direction of future changes. For these reasons, it was decided to rely upon parity distribution time series for the projection of those cohorts who have yet to enter childbearing age.

---

(4) A variant of this approach is to take into account series on cohort age-specific or cohort cumulative fertility rates.

TABLE 3. Implied Completed Fertility Rate Under Two Assumptions of Age Pattern of Fertility

| Cohort | Cumulative fertility up to indicated age per 1,000 women | | | Total fertility rate per 1,000 women derived from cumulative fertility up to indicated age and assuming .... | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Older age pattern | | | Younger age pattern | | |
| | 20 | 22 | 25 | 20 | 22 | 25 | 20 | 22 | 25 |
| 1939-40 ............ | 498 | 966 | 1,656 | 2,971 | 2,972 | 2,971 | 2,931 | 2,932 | 2,930 |
| 1940-41 ............ | 507 | 973 | 1,609 | 2,914 | 2,912 | 2,912 | 2,851 | 2,849 | 2,849 |
| 1941-42 ............ | 492 | 933 | 1,509 | 2,805 | 2,806 | 2,808 | 2,719 | 2,720 | 2,729 |
| 1942-43 ............ | 489 | 899 | 1,443 | 2,740 | 2,739 | 2,708 | 2,636 | 2,635 | 2,626 |
| 1943-44 ............ | 461 | 817 | 1,328 | 2,664 | 2,664 | 2,546 | 2,504 | 2,504 | 2,488 |
| 1944-45 ............ | 412 | 728 | | 2,600 | 2,603 | | 2,421 | 2,424 | |
| 1945-46 ............ | 394 | 699 | | 2,578 | 2,458 | | 2,396 | 2,344 | |
| 1946-47 ............ | 413 | 722 | | 2,601 | 2,480 | | 2,395 | 2,336 | |
| 1947-48 ............ | 371 | | | 2,510 | | | 2,320 | | |
| 1948-49 ............ | 351 | | | 2,440 | | | 2,260 | | |

Three measurements are directly related to parity distribution, and should be briefly defined for the sake of the reader unfamiliar with demographic concepts. The first such measurement is the completed fertility rate for each birth order. It is obtained by cumulating the birth order-age-specific rates up to age 49. The latter rates are constructed by dividing the births of each successive order, occurring to women in each age group, by the number of women in each corresponding age group. Thus, for the 1931-32 cohort, the completed fertility rate is 93.1 for the first birth order and 80.3 (per 100 women 49 years of age) for the second birth order. By adding together completed fertility rates by birth order, one gets the completed fertility rate for all birth orders, that is, the number of children born by a woman during her childbearing life. The second measurement is called the parity progression ratio, i.e. the proportion of women who have given birth to a child of the next higher order. It can also be viewed as the probability that a woman having "n" children will give birth to an additional child. The third measurement is the percentage distribution of women having exactly zero, one, two ... and "n" children. All these measurements are mathematically interrelated. Let $m_0$, $m_1$ ... $m_n$ and $m_{n+1}$ be the proportion of women with zero, one ... n and n+1 children, respectively; then the probability of having an additional child is given as follows:

$$a_0 = \frac{m_1}{m_0}$$

$$a_n = \frac{m_{n+1}}{m_n}$$

By multiplying and adding the conditional probabilities from $a_0$ to $a_n$ the Completed Fertility Rate per woman (all birth orders) is obtained:

$$CFR = a_0 + (a_0 \cdot a_1) \ldots + (a_0 \cdot a_1 \cdot \ldots a_n)$$

and by taking the differences between above probabilities, one obtains the proportion of women with exactly zero, one, two, n children:

$1.00 - a_0$ = the proportion of women having zero children;

$a_0 - (a_0 . a_1)$ = the proportion of women having one child;

$(a_0 . a_1) - (a_0 . a_1 . a_2)$ = the proportion of women having 2 children, and so on.

The two first measurements of parity distribution as defined above are presented for selected cohorts in Table 4. Charts 2 and 3 illustrate historical trends. Note in particular, the behaviour of the parity progression ratios: the very rapid climbing of the zero parity progression ratio, $a_0$ , and its asymptotic leveling-off for recent cohorts; a clear-cut downward tendency for progression ratios of third and higher parities; and finally, a moderately stable trend exhibited by $a_1$ and $a_2$. Later

we shall return to the analysis of historical trends in parity distribution and underlying factors but first a word has to be said about the general strategy to be followed in making projections for the cohorts considered in this sub-section.

Projections are made in terms of the completed fertility rates for each individual birth order. These are then added to obtain the over-all completed fertility rate, i.e. the number of children per woman (all birth orders) 49 years old. For the sake of simplicity and to avoid time-consuming numerical manipulation, efforts will be focused on the 1960-61 cohort, the selected target of this projection procedure, and not on all cohorts. Values of the over-all completed fertility rates for the intermediate cohorts, those born between 1948-49 and 1960-61, are derived by graphical interpolation.

The main effort here is directed toward the first birth order. This is so because of the compound effect that first birth order fertility has on subsequent childbearing probabilities, and by implication, on the completed fertility rate. Secondly, unlike higher parities, the determining variables can be more easily identified in the case of the first birth order.

Time series relevant to the fertility rate of the first birth order and other related factors have been drawn together in Chart 4 and will be referred to in subsequent discussion. From a relative low of 62.5 children, for the 1906-07 cohort, the first order completed fertility rate rose to 93.1 children per 100 women (49 years old) for the 1931-32 cohort. To encompass more recent cohorts, the cumulative fertility rates up to age 20 were calculated and shown on Chart 4 (curve 4). Fertility dropped rather sharply among the cohorts born between 1939 and 1949, though it appears that a resumption of the fertility trend is imminent.(5)

To formulate some sound assumptions about the future, we shall examine the factors behind past trends in first birth order fertility, and their probable impact in the years to come. Nuptiality seems to have played a dominant role in shaping the behaviour of the first order fertility rate, and can be expected to do so in the future. In Chart 4 nuptiality is represented by curve 2 (cumulative nuptiality rate to age 34, which for practical purposes could be considered the final nuptiality rate) and by curve 3 (cumulative nuptiality rate up to age 20). These two curves appear as a mirror image of the two curves reflecting the first birth order rates, thus underscoring the likelihood of a close causal relationship between nuptiality and fertility of the first birth order. Using cross-sectional analysis, Henripin has estimated that three quarters of the increase in the general fertility rate for Canada between 1941 and 1951 was due to an increase in the proportion of women married at various ages (Henripin, 1968). Similar estimates were made by George and Lapierre (1969) for the same period.

From a projection standpoint it is important to explain the likely causes of the recent reduction in the female nuptiality rate. Some writers attribute this phenomenon to extended education and to increasing difficulties encountered by young people in

(5) In view of the fact that for the most recent cohorts considered here, rates are partly based on the extrapolation of incomplete fertility, this decline should be interpreted with caution.

TABLE 4. Number of Children per 100 Women 49 Years of Age by Birth Order

| Birth order | Cohort | | | |
|---|---|---|---|---|
| | 1911-12 | 1921-22 | 1931-32 | 1937-38 |
| 1 | 69.3 | 84.9 | 93.1 | 90.8 |
| 2 | 56.4 | 73.5 | 80.3 | 76.5 |
| 3 | 40.4 | 53.4 | 59.8 | 52.9 |
| 4 | 27.7 | 35.6 | 37.4 | 29.3 |
| 5 | 19.0 | 22.8 | 22.4 | 14.0 |
| 6+ | 46.8 | 48.0 | 35.8 | 18.8 |
| Total | 259.6 | 318.2 | 328.8 | 282.3 |

TABLE 4(a). Parity Progression Ratio for Selected Cohorts

| Parity progression ratio | Cohort | | | |
|---|---|---|---|---|
| | 1911-12 | 1921-22 | 1931-32 | 1937-38 |
| 0 | .694 | .849 | .931 | .908 |
| 1 | .813 | .866 | .863 | .843 |
| 2 | .716 | .727 | .744 | .691 |
| 3 | .686 | .666 | .626 | .555 |
| 4 | .686 | .639 | .600 | .477 |
| 5 | .694 | .647 | .587 | .521 |
| 6 | .722 | .669 | .556 | .631 |
| 7 | .722 | .680 | .598 | .435 |

CHART 2

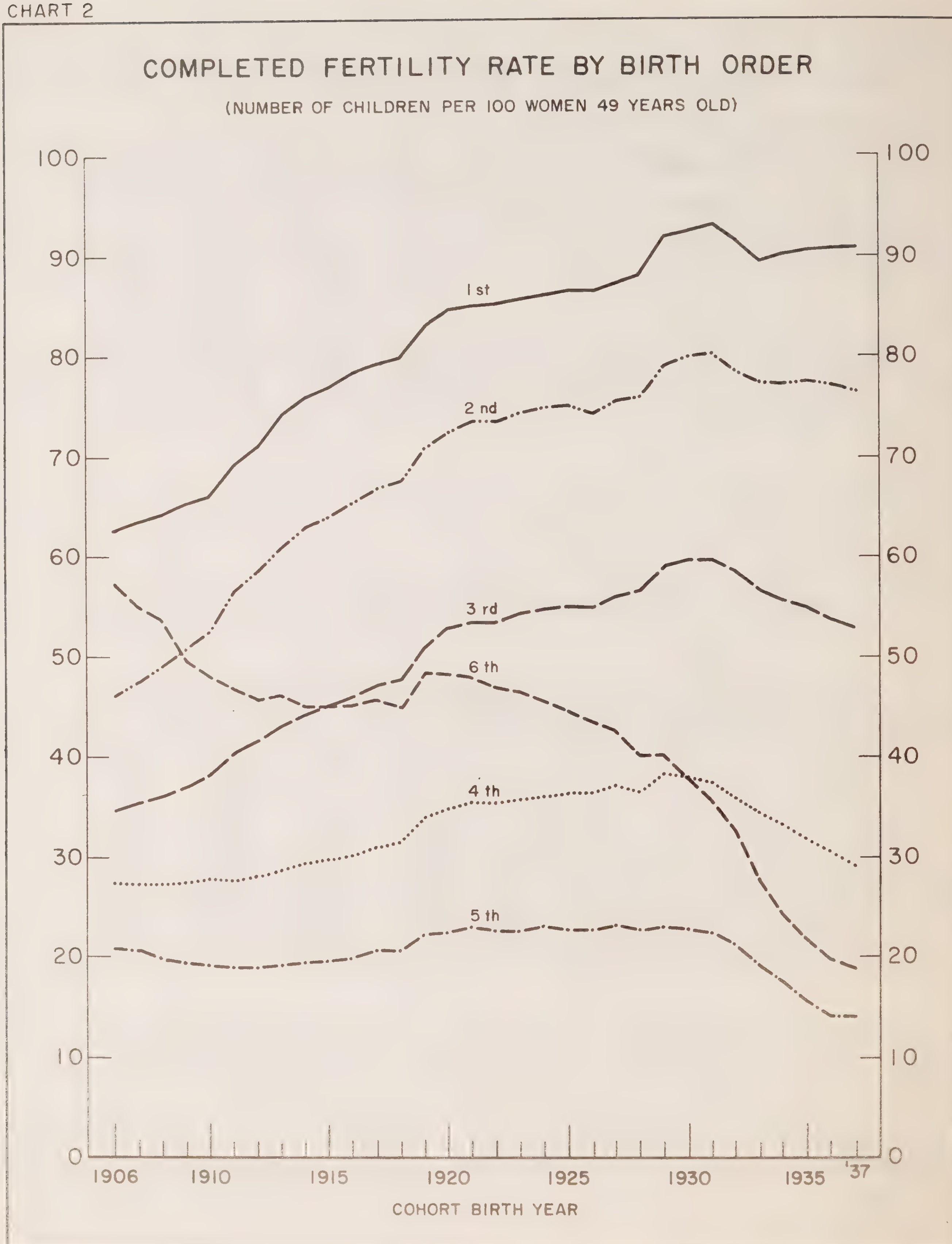

COMPLETED FERTILITY RATE BY BIRTH ORDER
(NUMBER OF CHILDREN PER 100 WOMEN 49 YEARS OLD)
100
90
80
70
60
50
40
30
20
10
0
1 st
2 nd
3 rd
6 th
4 th
5 th
1906
1910
1915
1920
1925
1930
1935
'37
COHORT BIRTH YEAR

CHART 3

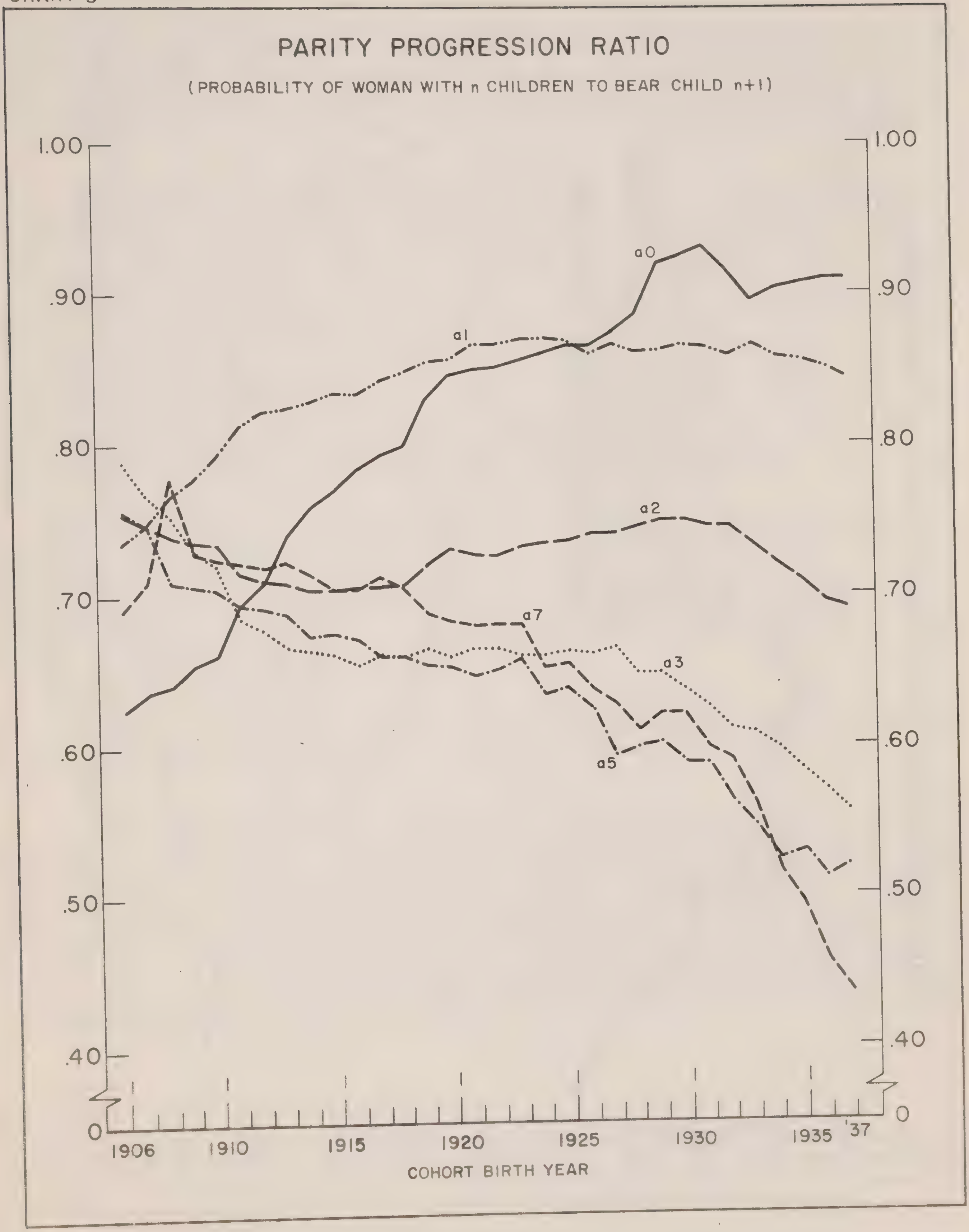
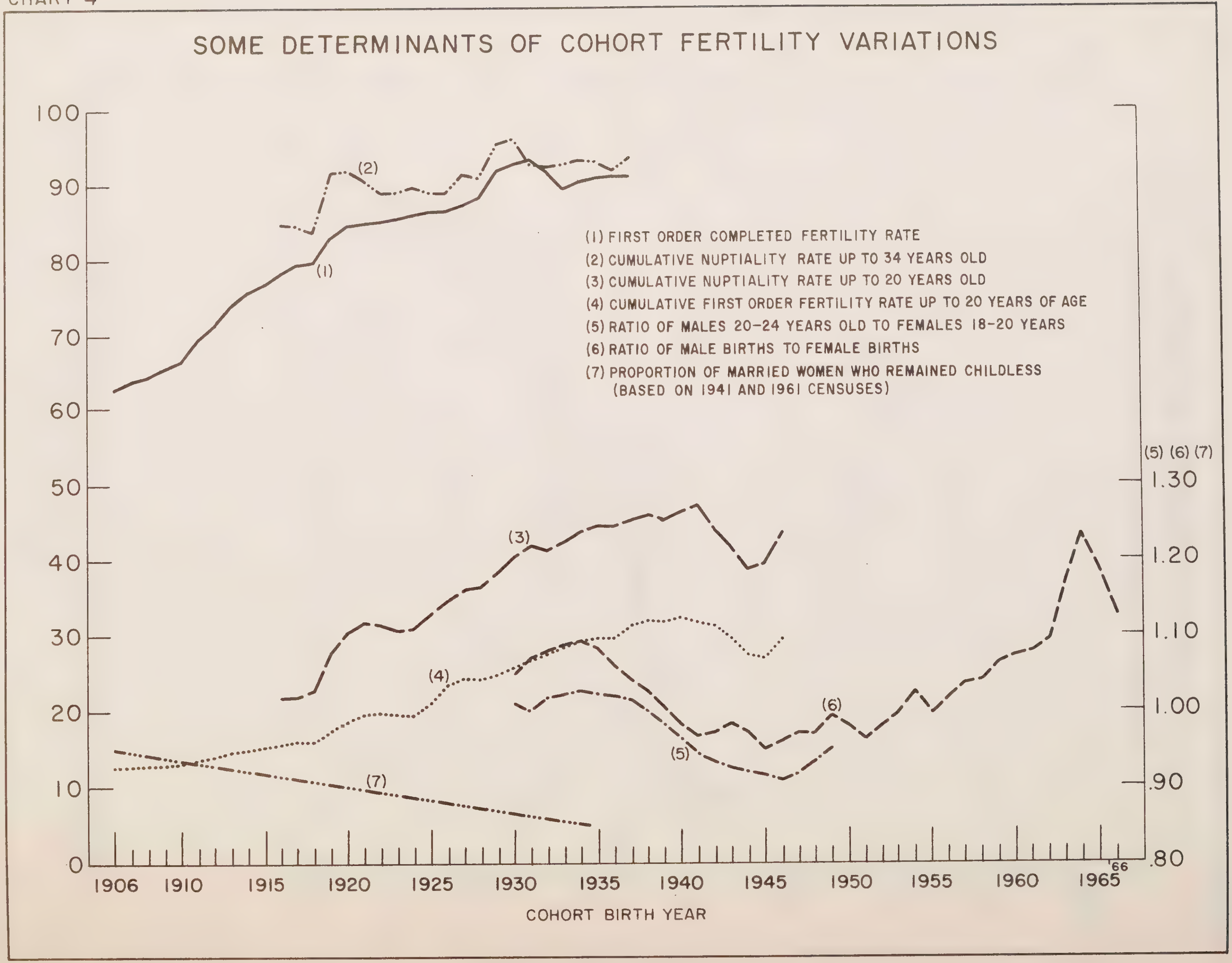

PARITY PROGRESSION RATIO
(PROBABILITY OF WOMAN WITH n CHILDREN TO BEAR CHILD n+1)
1.00
.90
.80
.70
.60
.50
.40
0
a0
a1
a2
a3
a5
a7
1906
1910
1915
1920
1925
1930
1935
'37
COHORT BIRTH YEAR

CHART 4


SOME DETERMINANTS OF COHORT FERTILITY VARIATIONS
(1) FIRST ORDER COMPLETED FERTILITY RATE
(2) CUMULATIVE NUPTIALITY RATE UP TO 34 YEARS OLD
(3) CUMULATIVE NUPTIALITY RATE UP TO 20 YEARS OLD
(4) CUMULATIVE FIRST ORDER FERTILITY RATE UP TO 20 YEARS OF AGE
(5) RATIO OF MALES 20-24 YEARS OLD TO FEMALES 18-20 YEARS
(6) RATIO OF MALE BIRTHS TO FEMALE BIRTHS
(7) PROPORTION OF MARRIED WOMEN WHO REMAINED CHILDLESS
(BASED ON 1941 AND 1961 CENSUSES)
(1)
(2)
(3)
(4)
(5)
(6)
(7)
100
90
80
70
60
50
40
30
20
10
0
(5) (6) (7)
1.30
1.20
1.10
1.00
.90
.80
1906
1910
1915
1920
1925
1930
1935
1940
1945
1950
1955
1960
1965
'66
COHORT BIRTH YEAR

finding jobs. Availability of more efficient contraceptives might have diminished the number of premarital pregnancies and hence the number of unplanned marriages (George and Lapierre, 1969). However, according to one theory, "changes in marriage rates and age at marriage in the last few years can be explained by the dynamics of the age and sex composition of the population, without recourse to extraneous factors for explanation" (Akers, 1967).

Prospects of integrating this theory at some stage in a population projection model prompts this writer to give it a more thorough consideration.(6) The theory can be briefly stated as follows. The customary practice is that men marry younger women. In Canada grooms are about two years older than their brides, that is to say, the latter belong to the cohorts born two years later than the former. When fertility is declining, the male cohorts will outnumber the female cohorts entering their respective ages of prime marriage, and the opposite will happen when fertility is rising. The latter situation is that of the cohorts born in the forties, being considered here. The excess of women over men, within the present arrangement with respect to age at marriage, leaves some women without partners, and this, according to the present theory, explains the observed decline in female nuptiality. The sex imbalance, while of little importance in a society where nuptiality is generally low, is likely to become a forceful inhibiting factor, depending on which sex outnumbers the other, when marriage tends to occur at early ages and to be almost universal.

Empirical evidence for Canada seems to support the above theory partially. To measure the marriage squeeze, we have calculated two ratios: one is the ratio of male births to female births two years later, the other is the ratio of males 20-25 to females 18-22 years old (represented on Chart 4 by curves 6 and 5, respectively). Though these curves differ in magnitude from those reflecting the nuptiality trends, they all exhibit some similarities with respect to the pattern of variation. In both cases the low point coincides with the same cohorts.

As children born during the period of declining fertility in the sixties reach marital age, the excess of females tends to disappear gradually with a possibility of male excess, thus providing more favourable demographic conditions for female nuptiality (see Chart 4, curve 6).

Other factors which contributed to the rise in the first birth order rate are the reduction in childlessness among married couples and the increase in the illegitimacy rate (Henripin, 1968). The proportion of married women remaining childless has dropped from about 15 per cent for the 1906-07 cohort to slightly above 5 per cent for the cohorts born in the 1930's (see curve 7, Chart 4). A further reduction in physiological sterility will be difficult to achieve but given the state of modern medical science it is not impossible. Furthermore, we do not anticipate a rise in voluntary childlessness. As Bumpass and Westoff have pointed out, "Virtually all American couples consider at least two children as desirable, and most achieve that number unless they are subfecund" (Bumpass and Westoff, 1969).

(6) The measurement of the marriage squeeze in terms of a ratio of male to female births can be calculated for the cohorts that will enter into prime marriage age only in the future. See curve 6 in Chart 4 representing this ratio for Canada.

A potent factor likely to affect the future fertility, in both its quantity and its timing, as Ryder and Westoff put it, is the increase in reproductive competence, whether by way of the development of better methods or the enlarged knowledge of available contraceptive procedures, or greater diligence in their employment (Ryder and Westoff, 1969). The incentive to avoid illegitimate conceptions is certainly strong enough to take advantage of improved contraceptives. Even among conjugal unions at least some first order pregnancies can be regarded as unwanted.

From the above review one may assume that the conditions for the maintenance of the present high level of first birth order fertility will continue to prevail. Except for the likelihood of the inhibiting effect of improved contraceptives, all the above-enumerated factors are expected to evolve in a direction favourable to such a high level. To provide a numerical range of variation, the upper level was set at 94 first births per 100 women, and the lower at 84 for the 1960-61 cohort.

Up to this point we have dealt with the first birth order. With this background subsequent parities can be handled swiftly. The second order birth rate is a function of the probability that women who already have one child will conceive a second. Significantly, this probability has varied within narrow limits, from about .82 to .86, for approximately the last 30 cohorts. There is no particular reason to think that there will be a considerable departure from this range for the years to come. Assuming, therefore, that the two values hold as a range for the future and applying them to the projected fertility rate for the first birth order, we obtain second birth order rates of 69 and 81 for the 1960-61 target cohort.

The third birth order rate for the cohorts born prior to 1930 has displayed a moderately upward trend and for the cohorts born since then a downward trend (see Charts 2 and 3). Since the prevailing tendency is toward a small family size, the best guess that can be made under the circumstances is that this birth order rate will continue to decrease but that its pace is uncertain. To accommodate this uncertainty, a rather wide range of possible variation, namely 41 and 57 births per 100 women, has been assigned to the 1960-61 cohort.

For all higher birth orders the historical series exhibit a clear-cut downward trend (see Charts 2 and 3). A reversal is most unlikely given the generally observed convergency toward families of small size. By a straightforward graphical extrapolation the rate for birth orders 6 and above was set between upper and lower limits of 7 and 13 per 100, respectively, for the 1960-61 cohort. Fourth and fifth orders are expected to evolve in a similar fashion. The assumed upper and lower levels for the individual parities are presented in Table 5. By summing these parities the completed fertility is seen to vary from 2.20 to 2.85. It is significant that Henripin and Légaré in their projections of fertility for Quebec arrived independently at nearly identical figures (Henripin and Légaré, 1969).

TABLE 5. Actual and Projected Completed Fertility Rate by Birth Order for Selected Cohorts

(per 100 women 49 years old)

| Birth order | 1911-12 | 1921-22 | 1931-32 | 1936-37 | 1937-38 | 1960-61 | |
|---|---|---|---|---|---|---|---|
| | | | | | | Low | High |
| 1 ..................... | 69.4 | 84.9 | 93.1 | 90.75 | 90.75 | 84.0 | 94.0 |
| 2 ..................... | 56.4 | 73.5 | 80.3 | 77.80 | 76.50 | 69.0 | 81.0 |
| 3 ..................... | 40.4 | 53.4 | 59.8 | 53.73 | 52.88 | 41.0 | 57.0 |
| 4 ..................... | 27.7 | 35.6 | 37.4 | 30.62 | 29.32 | 15.0 | 28.0 |
| 5 ..................... | 19.0 | 22.8 | 22.4 | 14.05 | 13.98 | 6.0 | 12.0 |
| 6+ .................... | 46.8 | 48.2 | 35.8 | 19.85 | 18.85 | 7.0 | 13.0 |
| Total ............... | 260.0 | 318.0 | 329.0 | 286.00 | 282.00 | 220.0 | 285.0 |

The completed fertility rates for all cohorts obtained by various methods described in this section are presented in Table 6 and Chart 5. The two intermediate projections shown were obtained by dividing the distance between the upper and lower limits into approximately equal parts for the 1960-61 cohort. For the intermediate cohorts the level and the shape of curves are effected by some adjustments to take account of the period total fertility trends (see Section 6).

## 4. Projecting the Age Pattern of Fertility

Having developed a set of projections of completed fertility rates, the next step is to distribute them by age of women within the reproductive span in order to obtain age-specific fertility rates by cohort. Hence, what is required at this stage is an assumption as to the future changes in the age pattern of fertility. The age pattern of fertility is an important factor to be accounted for in a projection. In the short run, changes in the age pattern of childbearing have a direct impact on the variations in the number of annual births. It has been argued that these changes may even be the predominant source of future variations, rivaling in this respect changes in family size (Ryder, 1969). In the long run, whether women tend to procreate at older or at younger ages, i.e. whether the mean length of a generation tends to increase or to decrease, makes a difference in the natural growth of a population (Coale and Tye, 1961). Assumptions regarding future changes in the age pattern of fertility were instrumental in projecting the completed fertility rate of cohorts with incomplete childbearing experience (Group II treated in sub-section 3.4).(7)

(7) In this particular instance the process has been reversed; the age pattern is projected first, then the completed fertility is derived by applying the assumed percentage distribution of fertility by age to the actual fertility rate completed by a cohort to the date at which the projection is made.

TABLE 6. Actual and Projected Completed Fertility Rates for the Annual Cohorts, 1905-06 to 1968-69

| Birth year of women (June 1 to May 31) | Age in 1969 | Age in 1984 | Number of children born or expected to be born per 1,000 women aged 49 years |
|---|---|---|---|
| | | | Actual rate |
| 1905-06 | 63 | 78 | 2,879 |
| 1906-07 | 62 | 77 | 2,841 |
| 1907-08 | 61 | 76 | 2,832 |
| 1908-09 | 60 | 75 | 2,736 |
| 1909-10 | 59 | 74 | 2,762 |
| 1910-11 | 58 | 73 | 2,711 |
| 1911-12 | 57 | 72 | 2,720 |
| 1912-13 | 56 | 71 | 2,767 |
| 1913-14 | 55 | 70 | 2,873 |
| 1914-15 | 54 | 69 | 2,813 |
| 1915-16 | 53 | 68 | 2,885 |
| 1916-17 | 52 | 67 | 2,879 |
| 1917-18 | 51 | 66 | 2,929 |
| 1918-19 | 50 | 65 | 2,893 |
| 1919-20 | 49 | 64 | 3,231 |
| | | | Projected rate |
| Group I: | | | |
| 1920-21 | 48 | 63 | 3,267 |
| 1921-22 | 47 | 62 | 3,286 |
| 1922-23 | 46 | 61 | 3,220 |
| 1923-24 | 45 | 60 | 3,258 |
| 1924-25 | 44 | 59 | 3,289 |
| 1925-26 | 43 | 58 | 3,272 |
| 1926-27 | 42 | 57 | 3,254 |
| 1927-28 | 41 | 56 | 3,303 |
| 1928-29 | 40 | 55 | 3,287 |
| 1929-30 | 39 | 54 | 3,423 |
| 1930-31 | 38 | 53 | 3,417 |
| 1931-32 | 37 | 52 | 3,414 |
| 1932-33 | 36 | 51 | 3,327 |
| 1933-34 | 35 | 50 | 3,232 |
| 1934-35 | 34 | 49 | 3,213 |
| 1935-36 | 33 | 48 | 3,185 |
| 1936-37 | 32 | 47 | 3,067 |
| 1937-38 | 31 | 46 | 3,058 |
| 1938-39 | 30 | 45 | 2,995 |

TABLE 6. Actual and Projected Completed Fertility Rates for the Annual Cohorts, 1905-06 to 1968-69 — Concluded

| Birth year of women (June 1 to May 31) | Age in 1969 | Age in 1984 | Number of children born or expected to be born per 1,000 women aged 49 years | | | |
|---|---|---|---|---|---|---|
| | | | Low assumption | Lower intermediate assumption | Upper intermediate assumption | High assumption |
| | | | Projected rate | | | |
| Group II: | | | | | | |
| 1939-40 ........... | 29 | 44 | 2,853 | 2,876 | 2,900 | 2,896 |
| 1940-41 ........... | 28 | 43 | 2,757 | 2,780 | 2,780 | 2,788 |
| 1941-42 ........... | 27 | 42 | 2,644 | 2,672 | 2,700 | 2,699 |
| 1942-43 ........... | 26 | 41 | 2,567 | 2,591 | 2,615 | 2,614 |
| 1943-44 ........... | 25 | 40 | 2,452 | 2,470 | 2,488 | 2,522 |
| 1944-45 ........... | 24 | 39 | 2,382 | 2,391 | 2,400 | 2,431 |
| 1945-46 ........... | 23 | 38 | 2,341 | 2,346 | 2,352 | 2,361 |
| 1946-47 ........... | 22 | 37 | 2,330 | 2,339 | 2,348 | 2,354 |
| 1947-48 ........... | 21 | 36 | 2,310 | 2,331 | 2,353 | 2,357 |
| 1948-49 ........... | 20 | 35 | 2,272 | 2,315 | 2,358 | 2,378 |
| Group III: | | | | | | |
| 1949-50 ........... | 19 | 34 | 2,260 | 2,316 | 2,372 | 2,385 |
| 1950-51 ........... | 18 | 33 | 2,246 | 2,311 | 2,377 | 2,384 |
| 1951-52 ........... | 17 | 32 | 2,236 | 2,318 | 2,400 | 2,460 |
| 1952-53 ........... | 16 | 31 | 2,227 | 2,326 | 2,425 | 2,488 |
| 1953-54 ........... | 15 | 30 | 2,219 | 2,335 | 2,450 | 2,523 |
| 1954-55 ........... | 14 | 29 | 2,212 | 2,341 | 2,470 | 2,569 |
| 1955-56 ........... | 13 | 28 | 2,208 | 2,351 | 2,495 | 2,650 |
| 1956-57 ........... | 12 | 27 | 2,206 | 2,358 | 2,510 | 2,715 |
| 1957-58 ........... | 11 | 26 | 2,205 | 2,363 | 2,520 | 2,770 |
| 1958-59 ........... | 10 | 25 | 2,200 | 2,365 | 2,530 | 2,810 |
| 1959-60 ........... | 9 | 24 | 2,200 | 2,370 | 2,540 | 2,844 |
| Group IV: | | | | | | |
| 1960-61 ........... | 8 | 23 | 2,190 | 2,370 | 2,550 | 2,850 |
| 1961-62 ........... | 7 | 22 | 2,190 | 2,370 | 2,550 | 2,850 |
| 1962-63 ........... | 6 | 21 | 2,190 | 2,370 | 2,550 | 2,850 |
| 1963-64 ........... | 5 | 20 | 2,190 | 2,370 | 2,550 | 2,850 |
| 1964-65 ........... | 4 | 19 | 2,190 | 2,370 | 2,550 | 2,850 |
| 1965-66 ........... | 3 | 18 | 2,190 | 2,370 | 2,550 | 2,850 |
| 1966-67 ........... | 2 | 17 | 2,190 | 2,370 | 2,550 | 2,850 |
| 1967-68 ........... | 1 | 16 | 2,190 | 2,370 | 2,550 | 2,850 |
| 1968-69 ........... | 0 | 15 | 2,190 | 2,370 | 2,550 | 2,850 |

Sources: The actual rates are derived from statistics on annual births by age of mother published in DBS, Vital Statistics (Catalogue No. 84-202); and Estimated Population of Canada by Provinces (Catalogue No. 91-201). For projected rates, see text.

CHART 5


ACTUAL AND PROJECTED
COMPLETED FERTILITY RATES BY COHORT
(NUMBER OF BIRTHS PER WOMAN 49 YEARS OLD)
ACTUAL RATES
PROJECTED RATES
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
0
1911
1915
1920
1925
1930
1935
1940
1945
1950
1955
1960
COHORT BIRTH YEAR

For the purpose of analysing and projecting the age pattern of fertility, a number of related measurements had to be calculated. These are: (a) the percentage distributions of fertility by 5-year age groups, (b) the median ages at (first) marriage and at maternity, for all births and for specific birth orders, and (c) median childspacing. The range of annual cohorts for which the required data are available or could be estimated covers the period 1916 to 1939. Table 7 brings together relevant estimates for selected cohorts.

TABLE 7. Percentage Distribution of Fertility by Age Group, and Some Related Indices, for the Selected Cohorts

| Age group | Cohort | | |
|---|---|---|---|
| | 1916-17 | 1930-31 | 1937-38 |
| | Percentage distribution of fertility | | |
| 14-19 | 4.59 | 6.40 | 9.34 |
| 20-24 | 21.61 | 30.26 | 37.28 |
| 25-29 | 29.56 | 32.69 | 29.28 |
| 30-34 | 24.14 | 20.38 | 14.07 |
| 35-39 | 14.87 | 8.33 | 7.44 |
| 40-44 | 4.89 | 2.38 | 2.38 |
| 45-49 | 0.34 | 0.21 | 0.21 |
| Total fertility rate | 2.879 | 3.449 | 3.128 |
| | in years | | |
| Median age at 1st birth | 24.36 | 22.58 | 21.86 |
| " " " 2nd " | 26.96 | 25.02 | 24.03 |
| " " " all births | 28.53 | 26.39 | 24.80 |
| " " " first marriage | 22.82 | 21.12 | 20.54 |
| " interval between 1st marriage and 1st birth | 1.54 | 1.50 | 1.32 |
| Median interval between 1st marriage and all births | 5.71 | 5.37 | 4.26 |
| Median interval between 1st and 2nd birth | 2.60 | 2.52 | 2.17 |
| | per 1,000 women | | |
| Cumulative nuptiality rate up to age 20 | 213.30 | 403.00 | 458.00 |

The data in Table 7 and the series in Charts 6 and 7 reveal a clear-cut tendency for women to have children at progressively younger ages. The median age of maternity dropped from 28.53 for the 1916-17 cohort to 24.80 for the 1937-38 cohort; the proportion of children born to these cohorts before reaching age 25 to all children born has increased from 26 to 46 per cent, respectively. These trends are reflections of three factors that can be singled out on the basis of the available statistics: nuptiality age pattern, childspacing, and parity distribution. Thus, the median age at the time of first marriage decreased from 22.82 to 20.54 and the median interval between age at first marriage and age at maternity dropped from 5.71 to 4.26. Women with 6 and more children constituted 15.8 per cent of all those with completed fertility for the 1916-17 cohort, while this percentage fell to 6.9 for the 1937-38 cohort. By contrast, the proportion of women with the two first birth orders jumped from 50.4 to 58.7 per cent, respectively.

The close association between the age pattern of fertility and its determinants — age pattern of nuptiality, parity distribution and childspacing — can also be inferred from the high zero-order correlation shown in Table 8.

TABLE 8. Zero-Order Correlation Between Relevant Indices of the Age Pattern of Fertility and its Determinants, for Cohorts Born Between 1916-17 and 1938-39

| | $X_0$ | $X_1$ | $X_2$ | $X_3$ |
|---|---|---|---|---|
| $X_0$ ...................... | — | | | |
| $X_1$ ...................... | .942 | — | | |
| $X_2$ ...................... | — .967 | — .850 | — | |
| $X_3$ ...................... | — .928 | — .859 | .910 | — |

Where $X_0$ is the proportion of births for the 14 to 24 age group, as an index of age pattern of fertility.
$X_1$ is the cumulative nuptiality up to age 20 as an index of the nuptiality age pattern.
$X_2$ is the mean parity of order 6 and over, as an index of parity distribution.
$X_3$ is the median interval between first and second births as an index of childspacing.

On the basis of the high correlation noted in Table 8, a two-stage model may be considered as suitable for projecting future trends in the age pattern. The first stage involves the projection of the determining variables. The second stage consists of deriving the age pattern of fertility, as a function of these determining variables.

CHART 6

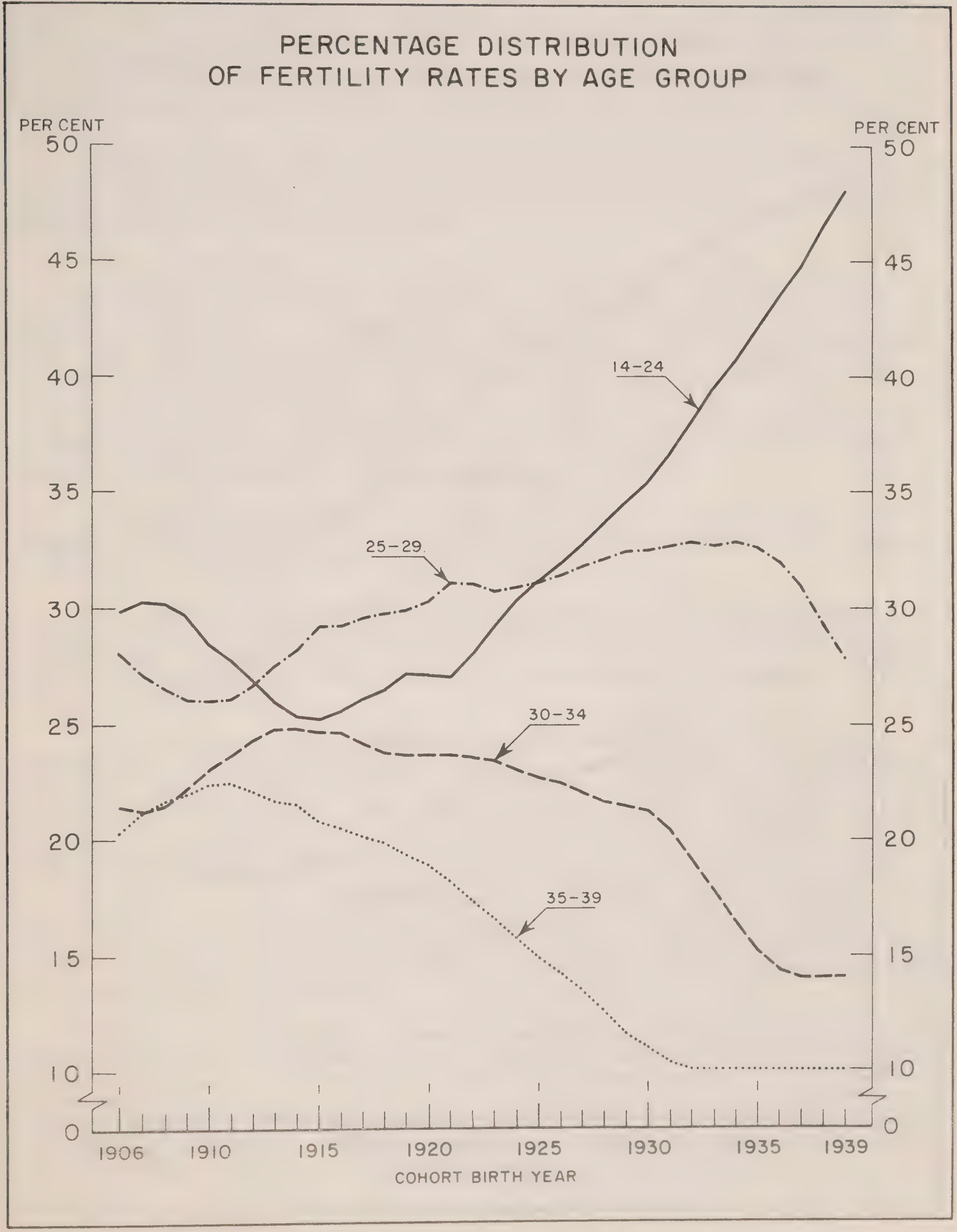

PERCENTAGE DISTRIBUTION
OF FERTILITY RATES BY AGE GROUP
PER CENT
PER CENT
50
45
40
35
30
25
20
15
10
0
14-24
25-29
30-34
35-39
1906
1910
1915
1920
1925
1930
1935
1939
COHORT BIRTH YEAR

CHART 7


MEDIAN AGE AT FIRST MARRIAGE
AND AT FIRST, SECOND AND ALL BIRTH ORDERS
AGE
29
28
27
26
25
24
23
22
21
20
0
ALL ORDERS
2nd ORDER
1st ORDER
1st MARRIAGE
1917
1920
1925
1930
1935
1940
1944
COHORT BIRTH YEAR

variables. The latter operation requires that the relationship between independent and dependent variables be mathematically defined. Currently, a multiple regression model is being tested and, if the results are satisfactory, it should have the capability of generating the necessary percentage distribution of fertility by age as a function of assumed levels of nuptiality, childspacing and parity distribution.

In the meantime, for the 1969 DBS population projection, the age pattern of fertility was projected by extrapolating past trends so as to take into account the prospective changes in nuptiality, age spacing and parity distribution. The following three assumptions were made in this respect.

The first assumption is that fertility will decrease further, both at the older and at the younger ages. While the reduction at older ages is seen as a logical sequence of the decrease in higher parities (see sub-section 3.5), the reduction at the youngest ages could occur if some delay in marriage were to take place. We have already noticed, while discussing the nuptiality trends in sub-section 3.5, that a drop in nuptiality among the cohorts born between 1940 and 1949 may be interpreted as being at least partly the consequence of the postponement of marriages.(8) The ensuing distribution takes a leptokurtic form, implying a shift in the burden of reproduction toward the central rather than the extreme reproductive ages. For purposes of identification, this assumption will be designated as assumption "A".

The second assumption "B" is that the past trend toward a younger mean age of maternity will increase yet as a result of a further downward shift in the parity distribution and of the lowering of age at marriage.

For similar reasons assumption "C" postulates a progressively younger age pattern of maternity as does the previous assumption, but in this case the process is expected to evolve at an accelerated pace. The more pronounced positive skewness is the peculiarity of the distribution implicit in this assumption as compared to the one inherent in assumption "B".

The validity of the latter two assumptions may be questioned by some on the grounds that the mean age at maternity for Canadian women is already quite low, and that a further downward shift will be hard to achieve. However, the American example demonstrates that still younger ages at marriage and at maternity are quite possible. Canadian females are lagging behind their American counterparts in terms of the various indices of the age pattern of nuptiality and fertility as a comparative study by Jacques Henripin and Nathan Keyfitz has revealed (Henripin, 1968; Henripin and Keyfitz, 1965). In particular, rates for Quebec females are far from those of American females in this respect. It is not unreasonable to assume that Canadians may well emulate Americans with respect to both younger age at marriage and shorter childspacing along with a further reduction in higher parities, all of which would contribute to a younger than ever age pattern of fertility.

Table 9 shows actual and assumed percentage distributions of fertility by five-year age groups for selected cohorts. The single-year age distribution within each five-year group was obtained pro rata the single age distribution observed for the 1937-38 cohort.

(8) For this reason it is thought that this assumption is particularly representative of the age pattern likely to characterize the fertility of the cohorts born between 1940 and 1949.

TABLE 9. Actual and Assumed Percentage Distribution of the Completed Fertility Rate, by Age, for Selected Cohorts, 1921-22 to 1960-61

| Age | 1921-22 | 1931-32 | 1937-38 | 1945-46 | | 1950-51 | | | 1960-61 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Pattern(1) | | Pattern | | | Pattern | | |
| | | | | A | B | A | B | C | A | B | C |
| 14-19 | 4.7 | 6.6 | 9.3 | 10.0 | 11.8 | 9.5 | 9.4 | 9.3 | 9.4 | 11.1 | 13.0 |
| 20-24 | 23.4 | 31.4 | 37.3 | 33.0 | 34.5 | 34.0 | 35.8 | 37.8 | 34.9 | 37.4 | 38.0 |
| 25-29 | 31.0 | 32.8 | 29.3 | 30.6 | 29.6 | 31.2 | 30.1 | 29.8 | 32.1 | 29.1 | 28.0 |
| 30-34 | 23.6 | 19.2 | 14.1 | 16.3 | 15.5 | 16.1 | 15.9 | 14.4 | 15.4 | 15.8 | 14.1 |
| 35-39 | 13.3 | 7.4 | 7.4 | 7.7 | 6.9 | 7.1 | 6.5 | 6.5 | 6.4 | 4.9 | 5.0 |
| 40-44 | 3.8 | 2.4 | 2.4 | 2.2 | 2.5 | 1.9 | 2.1 | 2.0 | 1.7 | 1.6 | 1.8 |
| 45-49 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.1 | 0.1 | 0.1 |
| Total | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |

(1) Age pattern C is expected to emerge only for the cohorts born after 1945-46.

## 5. Pairing Assumptions of Completed Fertility and of Age Patterns

At this stage in the operation, the question arises how to combine the assumptions regarding completed fertility with those regarding the fertility age pattern. The specific manner in which they are combined will have a direct bearing on the age-specific fertility rates.

Historical evidence does not offer sufficient clues to a clear-cut solution to this question (Ryder, 1969). During the 1940's and 1950's the women in Canada showed a tendency to have children at younger ages and to achieve larger families than during the pre-war period. By contrast, during the era of high fertility in Europe, the average age of mothers was probably quite high. According to a hypothesis put forward by Akers, to achieve very high fertility there must be many high order births, and high order births occur primarily to older women. But a very low fertility rate may also mean an older average age at maternity, since very low fertility can only be achieved if marriage is delayed which in turn delays family formation (Akers, 1965). It is likely that early maternity and small families will be the dominant feature of reproductive behaviour in modern society. Other real or hypothetical examples can be postulated to demonstrate that both positive and negative associations between the level and the age pattern of childbearing are possible.

The problem deserves more intensive theoretical and empirical investigation than is possible within the purview of this paper. However, for the present projection, a negative association between the fertility level and its age pattern was deemed preferable. Accordingly, the relatively old age pattern (A) was linked to the low fertility assumption, the young age pattern (B) to the intermediate fertility, and the very young age pattern (C) to the high fertility assumptions. Nuptiality is the prime factor behind this combination. It was hypothesized earlier in this paper (sub-section 3.5) that if the fertility rate does increase in the future, this will be achieved through a rise in nuptiality at younger ages. Furthermore, any tendency toward a relatively large-size family to be attained within a relatively short time span will be reflected in the shorter inter-pregnancy intervals.

The remaining steps are straightforward procedures. By multiplying the assumed completed fertility rates by the percentage distribution of fertility by age, age-specific fertility rates by cohorts were obtained. The next operation was to rearrange the latter rates from their cohort into period setting. Finally, by multiplying the age-specific fertility rates for each calendar year by the female survivors for the corresponding ages, the annual number of projected births was computed. These births were distributed by sex by applying the average ratio of 1,055 male births per 1,000 female births during 1958-68 (for details see Nagnur, 1969).

Total fertility rates for the successive calendar years 1969 to 1984 are shown in Table 10 and portrayed in Chart 8.

## 6. A Note on Consistency Tests

Projection by the cohort method, such as described in this paper, gives rise to complications in achieving internal consistency between various segments of the projected fertility series or between projected and actual past fertility series. Attention will be drawn here to two possible types of inconsistency: (a) between actual age-specific fertility rates and derived age-specific fertility rates for cohorts with incomplete fertility; (b) between the total (period) fertility rate for the last actual year and that for the first projected year.

### 6.1 The First Type of Inconsistency

Three procedural steps were followed to obtain the cohort age-specific fertility rates: first, the projection of completed fertility rate; secondly, the projection of the percentage distribution of fertility by age (age pattern); and thirdly, the calculation of the cohort age-specific fertility rates by multiplying the total fertility rate (first step) by its percentage distribution (second step). For a cohort in the childbearing age at the beginning of the projection, the age-specific rates thus derived will deviate to a greater or lesser degree from the age-specific fertility rates actually achieved up to the age which the cohort had reached at the start of the projection. A simple measure of deviation is to calculate the ratio of projected to actual rates, taken either individually for each age or cumulatively up to the last age at which the actual rate is available. If this ratio departs considerably from unity, a thorough revision of the underlying assumptions will be necessary. Discrepancies may arise from the failure to consistently project either the completed fertility rate and/or its age pattern. Small inconsistencies of this type can be taken care of by the formula below.

TABLE 10. Actual and Projected Period Total Fertility Rates per 1,000 Women, 1926-84

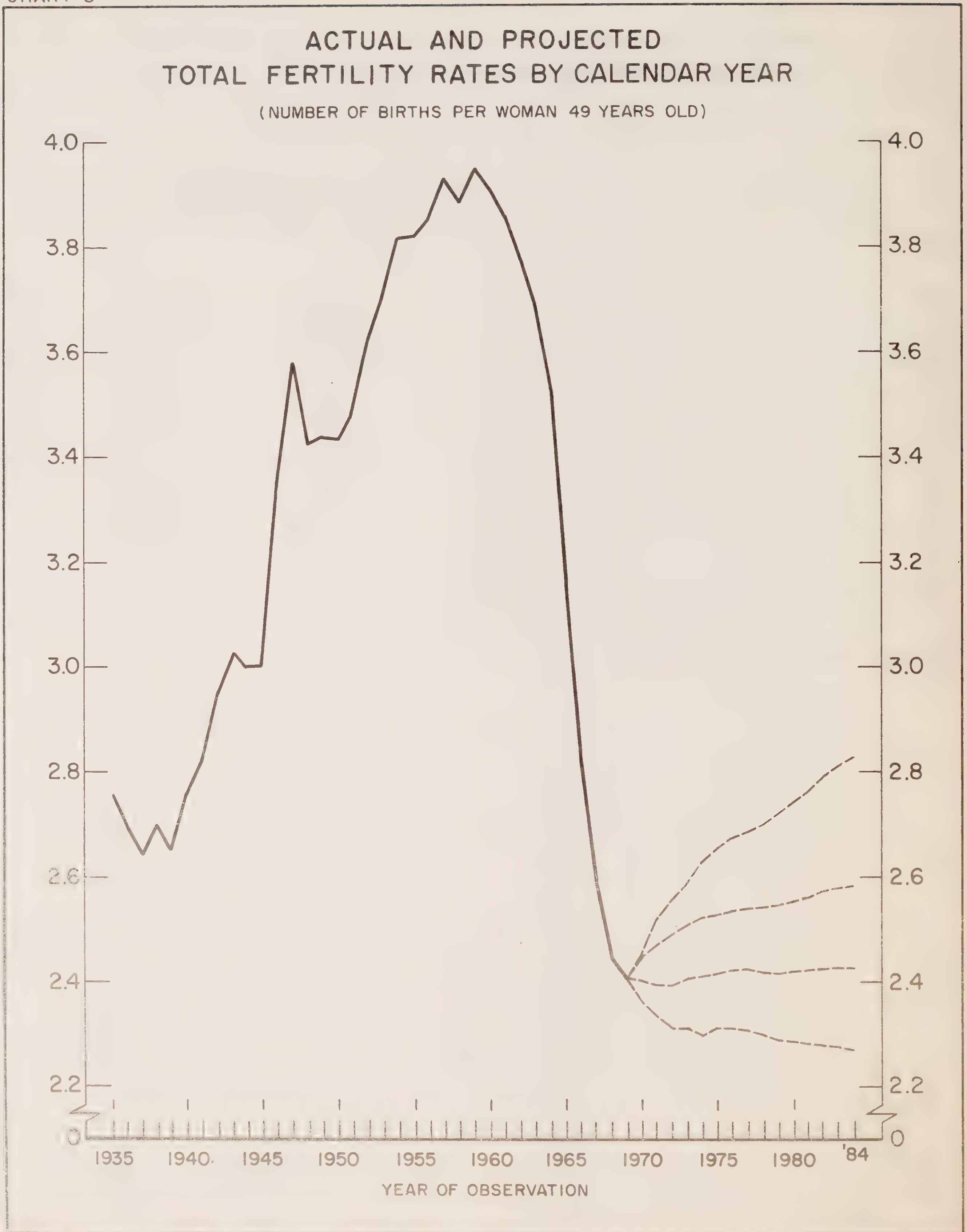
| Year | Actual rate |
|---|---|
| 1926 .......... | 3,356 |
| 1927 .......... | 3,319 |
| 1928 .......... | 3,296 |
| 1929 .......... | 3,218 |
| 1930 .......... | 3,284 |
| 1931 .......... | 3,201 |
| 1932 .......... | 3,086 |
| 1933 .......... | 2,865 |
| 1934 .......... | 2,804 |
| 1935 .......... | 2,754 |
| 1936 .......... | 2,695 |
| 1937 .......... | 2,645 |
| 1938 .......... | 2,701 |
| 1939 .......... | 2,653 |
| 1940 .......... | 2,759 |
| 1941 .......... | 2,824 |
| 1942 .......... | 2,954 |
| 1943 .......... | 3,030 |
| 1944 .......... | 3,000 |
| 1945 .......... | 3,005 |
| 1946 .......... | 3,356 |
| 1947 .......... | 3,575 |
| 1948 .......... | 3,423 |
| 1949 .......... | 3,438 |
| 1950 .......... | 3,433 |
| 1951 .......... | 3,480 |
| 1952 .......... | 3,621 |
| 1953 .......... | 3,702 |
| 1954 .......... | 3,812 |
| 1955 .......... | 3,817 |
| 1956 .......... | 3,849 |
| 1957 .......... | 3,929 |
| 1958 .......... | 3,884 |
| 1959 .......... | 3,947 |
| 1960 .......... | 3,910 |
| 1961 .......... | 3,857 |
| 1962 .......... | 3,773 |
| 1963 .......... | 3,690 |
| 1964 .......... | 3,521 |
| 1965 .......... | 3,163 |
| 1966 .......... | 2,826 |
| 1967 .......... | 2,593 |
| 1968 .......... | 2,445 |
| 1969 .......... | 2,410 |

TABLE 10. Actual and Projected Period Total Fertility Rates per 1,000 Women, 1926-84 — Concluded

| Year | Projected rate | | | |
|---|---|---|---|---|
| | Low assumption | Lower intermediate assumption | Upper intermediate assumption | High assumption |
| 1970 | 2,363 | 2,404 | 2,445 | 2,456 |
| 1971 | 2,334 | 2,399 | 2,464 | 2,519 |
| 1972 | 2,310 | 2,398 | 2,486 | 2,556 |
| 1973 | 2,310 | 2,409 | 2,508 | 2,584 |
| 1974 | 2,301 | 2,411 | 2,520 | 2,629 |
| 1975 | 2,312 | 2,419 | 2,526 | 2,651 |
| 1976 | 2,312 | 2,423 | 2,534 | 2,671 |
| 1977 | 2,310 | 2,424 | 2,538 | 2,681 |
| 1978 | 2,299 | 2,419 | 2,540 | 2,695 |
| 1979 | 2,286 | 2,415 | 2,544 | 2,715 |
| 1980 | 2,284 | 2,418 | 2,552 | 2,738 |
| 1981 | 2,280 | 2,420 | 2,559 | 2,762 |
| 1982 | 2,280 | 2,425 | 2,571 | 2,787 |
| 1983 | 2,275 | 2,426 | 2,576 | 2,808 |
| 1984 | 2,267 | 2,424 | 2,580 | 2,823 |

Source: Same as for Table 6.

CHART 8


ACTUAL AND PROJECTED
TOTAL FERTILITY RATES BY CALENDAR YEAR
(NUMBER OF BIRTHS PER WOMAN 49 YEARS OLD)
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
0
1935
1940
1945
1950
1955
1960
1965
1970
1975
1980
'84
YEAR OF OBSERVATION

This formula permits adjustment of the projected remaining fertility rates for a given cohort, so as to bring them into agreement on the one hand with the fertility rates for the ages for which they are known and, on the other hand, with the projected completed fertility rate.

$$a = \frac{C - A}{C - A'}$$

Where C = the <u>projected</u> completed fertility rate for a given cohort;
A = the <u>actual</u> fertility rate cumulated to age x (age up to which the rates are available);
A' = the <u>projected</u> fertility rate cumulated to the same age, respectively;
a = the factor by which the <u>projected</u> <u>remaining</u> age-specific fertility rates for that cohort have to be adjusted.

(The cumulative <u>actual</u> age-specific fertility rate up to age x plus the cumulative projected fertility rate from age x to the end of the childbearing span (49) should be equal after this adjustment to the projected completed fertility rate.)

## 6.2 <u>The Second Type of Inconsistency</u>

Under normal circumstances fertility rates vary smoothly from one year to the next and sudden shifts are unusual. Yet more often than not, one finds oneself with a series of total fertility rates as projected by the cohort method described in this paper which depart from the past series of actual total fertility rates to an extent unusual by any standard of empirical evidence. As Akers has put it, a possible flaw in the cohort fertility method is that "it does not provide for an even transition between past and future. Since it takes no particular account of the sequence of age-specific rates, it can produce sharp shifts in the level of fertility between the first year projected and the year preceding (Akers, 1965)". A total fertility rate that suddenly jumps by 15 per cent, for example, is an indication of some internal inconsistency in the projection.

This situation may arise from the failure to consistently project (a) completed fertility rates, (b) age pattern of fertility, or (c) failure to combine (a) and (b) properly. Unusual shifts may, however, appear even if the projection is basically "correct" on all three counts. It should be kept in mind that the total fertility rate for a given period is obtained by the summation of the age-specific fertility rates of 35 annual cohorts which have contributed births during that period. The formula below shows how the total fertility for 1970, for example, is related to the age-specific fertility rates of contributing cohorts. Even a small overstatement or understatement of the latter rates may generate by accumulation quite an impressive inflation or deflation of the period total fertility rate.

$$F_{1970} = {}_{1955}f_{15} + {}_{1954}f_{16} + {}_{1953}f_{17} \ldots + \ldots {}_{1921}f_{49}$$

or

$$= {}_{1955}D \cdot P_{15} + {}_{1954}D \cdot P_{16} + {}_{1953}D \cdot P_{17} \ldots + {}_{1921}D \cdot P_{49}$$

Where $F$ = the total period fertility rate for 1970 (first projected year);
$f_x$ = the fertility rate at age x for cohort y-x;
$D$ = the completed fertility rate for the cohort y-x; and
$P_x$ = the fertility rate at age x as a percentage of the completed fertility rate, for a given y-x cohort.

The above formula may be used to test the consistency between the actual total fertility rate for the last year and that for the first projected year. The analyst may well be advised to employ this check at the initial stage of projection to avoid lengthy calculations of series which could prove to be empirically unreasonable.

## 7. Summary and Concluding Comments

The fertility projections discussed in this report are based on the cohort approach. This method was preferred to the conventional period approach for two main reasons. First, by analysing and projecting fertility longitudinally in terms of real cohorts, the effects of changes in family size and in the age pattern of fertility on the annual total period fertility rate and the annual number of births can be duly assessed; secondly, this approach enables the analyst to take advantage of the child-bearing experience to date for projecting the remaining and final fertility rates of the cohort.

In projecting the final fertility rate, a distinction has been made between those cohorts with childbearing experience and those which have not yet reached the childbearing age at the beginning of the projection period. A number of methods have been attempted in projecting fertility from incomplete childbearing experience: arithmetical extrapolation based on the chain ratio method, fitting the Gompertz curve to that portion of fertility for which the actual data are available, graphical extrapolation of the birth order-age-specific fertility rates, and applying assumptions about the age pattern of fertility to the incomplete fertility for the cohorts for which the completed fertility rates are to be estimated.

There is scope for further research in devising methods that could help in the derivation of the final fertility rates for those cohorts with incomplete fertility. The Lawrence formula, referred to by Akers (1965) which states the remaining cohort fertility, at any age, as a function of the mean age of the mother over the remaining years, and the translation approach used by Ryder (1964) between cohort and period total fertility rates are cases in point. Another approach is to make use of surveys regarding the fertility expectations of women. Though the adequacy of these surveys as a basis for the projection of family size has been seriously challenged (Ryder and Westoff, 1967), with a proper sample of population and battery of questions on future expectations, they may well constitute a source of valuable information for projections (Siegel and Akers, 1969). Even so, Bumpass and Westoff (1969) in using longitudinal data from the Princeton Fertility Study arrived at the conclusion that the average number of children desired by women after the birth of their second child is a good predictor of the average completed family size.

Although a great deal of work remains to be done before a highly predictive model can be developed, incomplete cohort fertility, with the proper methodological developments and the addition of complementary information on related fertility variables,

is a promising basis for short-term projections. A real challenge is presented by the forecasting of fertility for a longer period. For this purpose a purely demographic model which could be used in the short term, becomes insufficient; other basic economic and social variables related to fertility have to be included. A socio-demographic model of this type remains an objective as yet to be achieved. A promising effort in this direction is a recent work by Denton and Spencer (1970).

As far as the projection of cohorts with no childbearing experience is concerned, the approach used was primarily the extrapolation of past trends. However, instead of using general fertility rates, series of parity distribution were used. These series exhibit a clear time perspective and offer the analytical advantage of making projections in terms of each individual birth order. Attempts have been made, particularly with the first birth order, to base the projections on prospective trends, particularly in nuptiality and childlessness.

The second component of fertility projections by the cohort method is the age pattern of fertility. Projections of this component are based on the extrapolation of past trends with some assumptions about the future movement of nuptiality, parity distribution and childspacing, the three determinants of the age structure of fertility.

Additional research is required to assess the effect on the cohort-age-specific fertility rates of the combination of assumptions regarding the completed fertility rates and those regarding the age pattern of fertility. In the present work we have associated a relatively young age pattern with a relatively high completed rate, and a relatively old age pattern with a relatively low completed rate, to generate cohort-age-specific fertility rates.

Finally, another area where additional research is necessary, is to find a simple method that would ensure a smooth transition from actual to projected series. Some of the related problems have been discussed in the final section of this report.

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